Name______________________________ Date__________

Chapter 6 Maintaining Mathematical Proficiency

Evaluate the expression.

1. $(14 + 20 - 6) \div 4 - 6^2$ **2.** $(8 + 4)^2 + (13 - 10 \div 5)$ **3.** $8 \div 4 \bullet 19 + 18 + 13$

4. $3 \bullet 14 \bullet 11 + 4^2 + 19$ **5.** $(21 + 2)(14 - 6) + 3^2$ **6.** $7(3 \bullet 10 - 4^2) + 8$

Evaluate the expression.

7. 64^0 **8.** 4^{-2} **9.** $(-3)^{-3}$ **10.** $7^0 + 5^{-2}$

11. $(-2)^{-6} \bullet 8^0$ **12.** $7^3 \bullet 7^{-3}$ **13.** $10^2 \div (-5)^{-2}$ **14.** $6^{-2} \div 1^9 \bullet 9$

Write an equation for the *n*th term of the arithmetic sequence.

15. $1, 5, 9, 13, \ldots$ **16.** $21, 15, 9, 3, \ldots$ **17.** $-2, 1, 4, 7, \ldots$

18. $8, 6, 4, 2, \ldots$ **19.** $-10, -4, 2, 8, \ldots$ **20.** $16, 8, 0, -8, \ldots$

Name ____ Date __________

6.1 Exponential Functions

For use with Exploration 6.1

Essential Question What are some of the characteristics of the graph of an exponential function?

1 EXPLORATION: Exploring an Exponential Function

Work with a partner. Complete each table for the *exponential function* $y = 16(2)^x$. In each table, what do you notice about the values of x? What do you notice about the values of y?

x	$y = 16(2)^x$
0	
1	
2	
3	
4	
5	

x	$y = 16(2)^x$
0	
2	
4	
6	
8	
10	

2 EXPLORATION: Exploring an Exponential Function

Work with a partner. Repeat Exploration 1 for the exponential function $y = 16\left(\frac{1}{2}\right)^x$.

x	$y = 16\left(\frac{1}{2}\right)^x$
0	
1	
2	
3	
4	
5	

x	$y = 16\left(\frac{1}{2}\right)^x$
0	
2	
4	
6	
8	
10	

Do you think the statement below is true for *any* exponential function? Justify your answer.

"*As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor*."

3 EXPLORATION: Graphing Exponential Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Sketch the graphs of the functions given in Explorations 1 and 2. How are the graphs similar? How are they different?

Communicate Your Answer

4. What are some of the characteristics of the graph of an exponential function?

5. Sketch the graph of each exponential function. Does each graph have the characteristics you described in Question 4? Explain your reasoning.

a. $y = 2^x$

b. $y = 2(3)^x$

c. $y = 3(1.5)^x$

d. $y = \left(\frac{1}{2}\right)^x$

e. $y = 3\left(\frac{1}{2}\right)^x$

f. $y = 2\left(\frac{3}{4}\right)^x$

Name ____ Date _________

6.1 Notetaking with Vocabulary

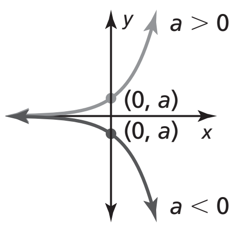
For use after Lesson 6.1

In your own words, write the meaning of each vocabulary term.

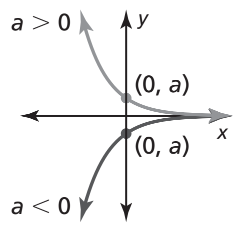
exponential function

Core Concepts

Graphing $y = ab^x$ When $b > 1$



Graphing $y = ab^x$ When $0 < b < 1$



Notes:

Extra Practice

In Exercises 1–4, determine whether the table represents an exponential function. Explain.

1.

x	*y*
1	8
2	4
3	2
4	1

2.

x	*y*
1	3
2	7
3	11
4	15

3.

x	*y*
–1	12
0	9
1	6
2	3

4.

x	*y*
–1	0.125
0	0.5
1	2
2	8

In Exercises 5–7, evaluate the function for the given value of *x*.

5. $y = 3^x; x = 5$

6. $y = \left(\frac{1}{4}\right)^x; x = 3$

7. $y = 3(4)^x; x = 4$

In Exercises 8 and 9, graph the function. Compare the graph to the graph of the parent function. Describe the domain and range of *f*.

8. $f(x) = -2^x$

9. $f(x) = 2\left(\frac{1}{4}\right)^x$

Name ____ Date __________

6.1 Notetaking with Vocabulary (continued)

In Exercises 10 and 11, graph the function. Describe the domain and range.

10. $f(x) = 4^x - 2$

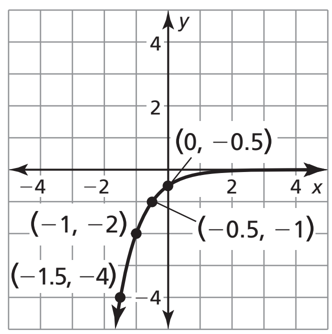
11. $f(x) = 4\left(\frac{1}{2}\right)^{x+1}$

In Exercises 12 and 13, write an exponential function represented by the table or graph.

12.

x	0	1	2	3
***f*(*x*)**	3	18	108	648

13.



14. Graph the function $f(x) = 2^x$. Then graph $g(x) = 2^x + 3$. How are the y-intercept, domain, and range affected by the translation?

Name____ Date__________

6.2 Exponential Growth and Decay

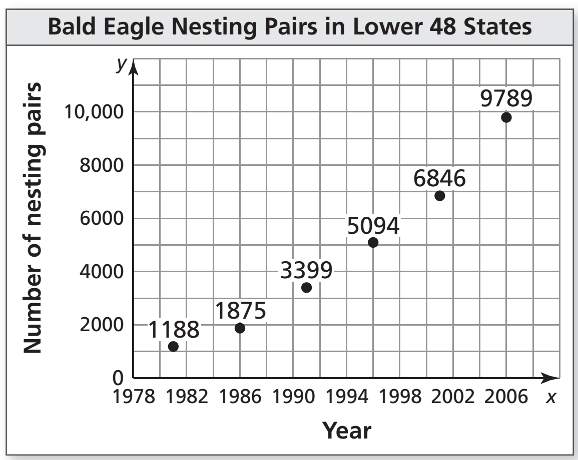
For use with Exploration 6.2

Essential Question What are some of the characteristics of exponential growth and exponential decay functions?

1 EXPLORATION: Predicting a Future Event

Work with a partner. It is estimated, that in 1782, there were about 100,000 nesting pairs of bald eagles in the United States. By the 1960s, this number had dropped to about 500 nesting pairs. In 1967, the bald eagle was declared an endangered species in the United States. With protection, the nesting pair population began to increase. Finally, in 2007, the bald eagle was removed from the list of endangered and threatened species.

Describe the pattern shown in the graph. Is it exponential growth? Assume the pattern continues. When will the population return to that of the late 1700s? Explain your reasoning.



2 EXPLORATION: Describing a Decay Pattern

Work with a partner. A forensic pathologist was called to estimate the time of death of a person. At midnight, the body temperature was 80.5°F and the room temperature was a constant 60°F. One hour later, the body temperature was 78.5°F.

a. By what percent did the difference between the body temperature and the room temperature drop during the hour?

b. Assume that the original body temperature was 98.6°F. Use the percent decrease found in part (a) to make a table showing the decreases in body temperature. Use the table to estimate the time of death.

Time (h)								
Temperature difference (°F)								
Body temperature (°F)								

Communicate Your Answer

3. What are some of the characteristics of exponential growth and exponential decay functions?

4. Use the Internet or some other reference to find an example of each type of function. Your examples should be different than those given in Explorations 1 and 2.

a. exponential growth

b. exponential decay

Name____ Date__________

6.2 Notetaking with Vocabulary
For use after Lesson 6.2

In your own words, write the meaning of each vocabulary term.

exponential growth

exponential growth function

exponential decay

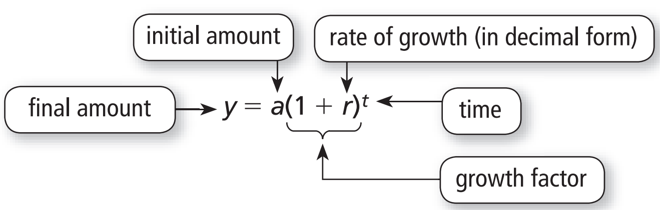
exponential decay function

compound interest

Core Concepts

Exponential Growth Functions

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$, is an **exponential growth function**.



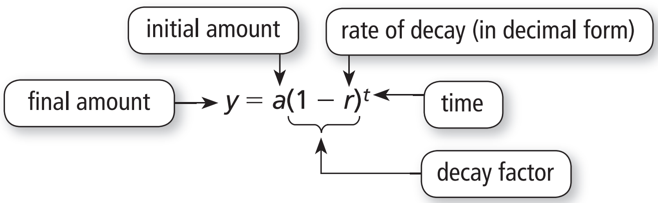
Notes:

Name ____ Date __________

6.2 Notetaking with Vocabulary (continued)

Exponential Decay Functions

A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$, is an **exponential decay function**.



Notes:

Compound Interest

Compound interest is the interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}.$$

P = principal (initial amount)

r = annual interest rate (in decimal form)

t = time (in years)

n = number of times interest is compounded per year

Notes:

Extra Practice

1. In 2005, there were 100 rabbits in Polygon Park. The population increased by 11% each year.

a. Write an exponential growth function that represents the population t years after 2005.

b. What will the population be in 2025? Round your answer to the nearest whole number.

In Exercises 2–5, determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain.

2.

x	*y*
0	20
1	30
2	45
3	67.5

3.

x	*y*
–1	160
0	40
1	10
2	2.5

4.

x	*y*
1	32
2	22
3	12
4	2

5.

x	*y*
–1	4
0	10
1	25
2	62.5

In Exercises 6–8, determine whether each function represents *exponential growth* or *exponential decay*. Identify the percent rate of change.

6. $y = 4(0.95)^t$

7. $y = 500(1.08)^t$

8. $w(t) = \left(\frac{3}{4}\right)^t$

In Exercises 9 and 10, write a function that represents the balance after *t* years.

9. \$3000 deposit that earns 6% annual interest compounded quarterly.

10. \$5000 deposit that earns 7.2% annual interest compounded monthly.

Name ____ Date __________

6.3 Comparing Linear and Exponential Functions

For use with Exploration 6.3

Essential Question How can you compare the growth rates of linear and exponential functions?

1 EXPLORATION: Comparing Values

Work with a partner. An art collector buys two paintings. The value of each painting after *t* year is *y* dollars. Complete each table. Compare the values of the two paintings. Which painting's value has a constant growth rate? Which painting's value has an increasing growth rate? Explain your reasoning.

t	$y = 19t + 5$
0	
1	
2	
3	
4	

t	$y = 3^t$
0	
1	
2	
3	
4	

2 EXPLORATION: Comparing Values

Work with a partner. Analyze the values of the two paintings over the given time periods. The value of each painting after t years is y dollars. Which painting's value eventually overtakes the other?

t	$y = 19t + 5$
4	
5	
6	
7	
8	
9	

t	$y = 3^t$
4	
5	
6	
7	
8	
9	

3 EXPLORATION: Comparing Graphs

Work with a partner. Use the tables in Explorations 1 and 2 to graph $y = 19t + 5$ and $y = 3^t$ in the same coordinate plane. Compares the graph of the functions.

Communicate Your Answer

3. How can you compare the growth rates of linear and exponential functions?

4. Which function has a growth rate that is eventually much greater than the growth rates of the other function? Explain your reasoning.

Name ____ Date __________

6.3 Notetaking with Vocabulary
For use after Lesson 6.3

In your own words, write the meaning of each vocabulary term.

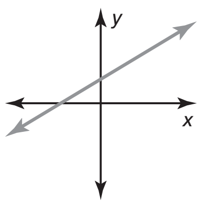
average rate of change

Core Concepts

Linear, Exponential, and Quadratic Functions

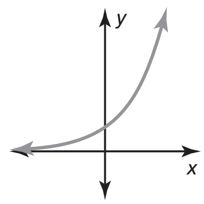
Linear Function

$$y = mx + b$$



Exponential Function

$$y = ab^x$$



Notes:

Differences and Ratios of Functions

You can use patterns between consecutive data pairs to determine which type of function models the data.

- **Linear Function** The differences of consecutive y-values are constant.
- **Exponential Function** Consecutive y-values have a common *ratio*.

In each case, the differences of consecutive x-values need to be constant.

Notes:

\Name ______________________________ Date __________

Comparing Functions Using Average Rates of Change

As a and b increase, the average rate of change between $x = a$ and $x = b$ of an increasing exponential function $y = f(x)$ will eventually exceed the average rate of change between $x = a$ and $x = b$ of an increasing linear function $y = g(x)$. So, as x increases, $f(x)$ will eventually exceed $g(x)$.

Notes:

Extra Practice

In Exercises 1–4, plot the points. Tell whether the points appear to represent a *linear function*, an *exponential function*, or *neither*.

1. $(-3, 2), (-2, 4), (-4, 4), (-1, 8), (-5, 8)$

2. $(-3, 1), (-2, 2), (-1, 4), (0, 8), (2, 14)$

3. $(4, 0), (2, 1), (0, 3), (-1, 6), (-2, 10)$

4. $(2, -4), (0, -2), (-2, 0), (-4, 2), (-6, 4)$

6.3 Notetaking with Vocabulary (continued)

In Exercises 5 and 6, tell whether the table of values represents a *linear*, or an *exponential* function.

5.

x	-2	-1	0	1	2
y	7	4	1	-2	-5

6.

x	-2	-1	0	1	2
y	$\frac{1}{18}$	$\frac{1}{3}$	2	12	72

In Exercises 7 and 8, tell whether the data represent a *linear*, or an *exponential* function. Then write the function.

7. $(-2, -4), (-1, -1), (0, 2), (1, 5), (2, 8)$

8. $(-2, 1.75), (-1, 3.5), (0, 7), (1, 14), (2, 28)$

9. A person invests \$1000 into an account that earns compound interest. The table shows the amount A (in dollars) in the account after t (in years) time has passed. Tell whether the data can be modeled by a *linear* or an *exponential* function. Explain.

Time, t	0	1	2	3	4
Amount, A	1000	1050	1102.50	1157.63	1215.51

Name____ Date__________

6.4 Solving Exponential Equations
For use with Exploration 6.4

Essential Question How can you solve an exponential equation graphically?

1 EXPLORATION: Solving an Exponential Equation Graphically

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

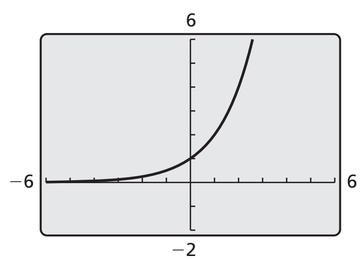
Work with a partner. Use a graphing calculator to solve the exponential equation $2.5^{x-3} = 6.25$ graphically. Describe your process and explain how you determined the solution.

2 EXPLORATION: The Number of Solutions of an Exponential Equation

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use a graphing calculator to graph the equation $y = 2^x$.



b. In the same viewing window, graph a linear equation (if possible) that does not intersect the graph of $y = 2^x$.

c. In the same viewing window, graph a linear equation (if possible) that intersects the graph of $y = 2^x$ in more than one point.

d. Is it possible for an exponential equation to have no solution? more than one solution? Explain your reasoning.

3 EXPLORATION: Solving Exponential Equations Graphically

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use a graphing calculator to solve each equation.

a. $2^x = \frac{1}{2}$

b. $2^{x+1} = 0$

c. $2^x = 1$

d. $3^x = 9$

e. $3^{x-1} = 0$

f. $4^{2x} = \frac{1}{16}$

g. $2^{3x} = \frac{1}{8}$

h. $3^{x+2} = \frac{1}{9}$

i. $2^{x-2} = \frac{3}{2}x - 2$

Communicate Your Answer

4. How can you solve an exponential equation graphically?

5. A population of 30 mice is expected to double each year. The number p of mice in the population each year is given by $p = 30(2^n)$. In how many years will there be 960 mice in the population?

Name____ Date__________

6.4 Notetaking with Vocabulary

For use after Lesson 6.4

In your own words, write the meaning of each vocabulary term.

exponential equation

Core Concepts

Property of Equality for Exponential Equations

Words Two powers with the *same positive base b*, where $b \neq 1$, are equal if and only if their exponents are equal.

Numbers If $2^x = 2^5$, then $x = 5$. If $x = 5$, then $2^x = 2^5$.

Algebra If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

Notes:

Extra Practice

In Exercises 1–15, solve the equation. Check your solution.

1. $3^{4x} = 3^{12}$

2. $8^{x+5} = 8^{20}$

3. $6^{4x-5} = 6^{2x}$

4. $5^{6x-3} = 5^{-3+4x}$

5. $4^{2x+11} = 1024$

6. $8^{3-2x} = 512$

7. $4^{7-x} = 256$

8. $49^{x-2} = 343$

9. $36^{6x-1} = 6^{5x}$

10. $9^{x-4} = 81^{3x}$

11. $64^{x+1} = 512^{x}$

12. $6^{2x} = 36^{2x+1}$

Name____ Date__________

6.4 Notetaking with Vocabulary (continued)

13. $\left(\frac{1}{7}\right)^{x} = 2401$

14. $\dfrac{1}{512} = 2^{3x-1}$

15. $25^{2-2x} = \left(\dfrac{1}{625}\right)^{x+1}$

In Exercises 16–21, use a graphing calculator to solve the equation.

16. $3^{x+3} = 9$

17. $\left(\dfrac{1}{4}\right)^{-x-1} = 64$

18. $-2x - 2 = -2^{-x+1}$

19. $2^{x+2} = 5^{x+2}$

20. $7^{-x+1} = 4^{x-1}$

21. $-\dfrac{1}{2}x - 3 = \left(\dfrac{2}{3}\right)^{2x-1}$

22. You deposit $1000 in a savings account that earns 5% annual interest compounded yearly.

a. Write an exponential equation to determine when the balance of the account will be $1500.

b. Solve the equation.

Name ____ Date __________

6.5 Geometric Sequences

For use with Exploration 6.5

Essential Question How can you use a geometric sequence to describe a pattern?

In a **geometric sequence**, the ratio between each pair of consecutive terms is the same. This ratio is called the **common ratio**.

1 EXPLORATION: Describing Calculator Patterns

Work with a partner. Enter the keystrokes on a calculator and record the results in the table. Describe the pattern.

a. Step 1 2 =

Step 2 × 2 =

Step 3 × 2 =

Step 4 × 2 =

Step 5 × 2 =

Step	1	2	3	4	5
Calculator display					

b. Step 1 6 4 =

Step 2 × . 5 =

Step 3 × . 5 =

Step 4 × . 5 =

Step 5 × . 5 =

Step	1	2	3	4	5
Calculator display					

c. Use a calculator to make your own sequence. Start with any number and multiply by 3 each time. Record your results in the table.

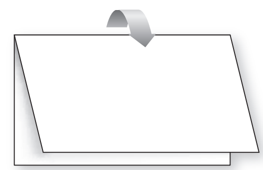
Step	1	2	3	4	5
Calculator display					

d. Part (a) involves a geometric sequence with a common ratio of 2. What is the common ratio in part (b)? part (c)?

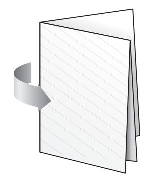
2 EXPLORATION: Folding a Sheet of Paper

Work with a partner. A sheet of paper is about 0.1 millimeter thick.

a. How thick will it be when you fold it in half once? twice? three times?



b. What is the greatest number of times you can fold a piece of paper in half? How thick is the result?



c. Do you agree with the statement below? Explain your reasoning.

"*If it were possible to fold the paper in half 15 times, it would be taller than you*."

Communicate Your Answer

3. How can you use a geometric sequence to describe a pattern?

4. Give an example of a geometric sequence from real life other than paper folding.

Name ____ Date __________

6.5 Notetaking with Vocabulary
For use after Lesson 6.5

In your own words, write the meaning of each vocabulary term.

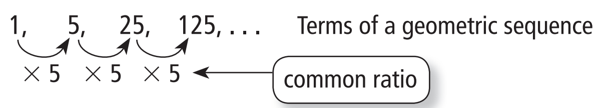
geometric sequence

common ratio

Core Concepts

Geometric Sequence

In a **geometric sequence**, the ratio between each pair of consecutive terms is the same. This ratio is called the **common ratio**. Each term is found by multiplying the previous term by the common ratio.



Notes:

Equation for a Geometric Sequence

Let a_n be the nth term of a geometric sequence with first term a_1 and common ratio r. The nth term is given by

$$a_n = a_1 r^{n-1}.$$

Notes:

Extra Practice

In Exercises 1–6, determine whether the sequence is *arithmetic*, *geometric*, or *neither*. Explain your reasoning.

1. $1, -4, 16, -64, \ldots$

2. $3, 7, 11, 15, \ldots$

3. $2, 4, 8, 32, \ldots$

4. $12, 9, 7, 5, \ldots$

5. $6, 18, 54, 162, \ldots$

6. $11, 19, 27, 35, \ldots$

In Exercises 7–9, write the next three terms of the geometric sequence.

7. $7, 21, 63, 189, \ldots$

8. $576, 288, 144, 72, \ldots$

9. $5, -10, 20, -40, \ldots$

In Exercises 10–12, write the next three terms of the geometric sequence. Then graph the sequence.

10. $12, 6, 3, \frac{3}{2}, \ldots$

11. $3, 12, 48, 192, \ldots$

12. $0.008, 0.04, 0.2, 1, \ldots$

6.5 Notetaking with Vocabulary (continued)

In Exercises 13–20, write an equation for the *n*th term of the geometric sequence. Then find a_6.

13. $6561, 2187, 729, 243, \ldots$ **14.** $8, -24, 72, -216, \ldots$ **15.** $3, 15, 75, 375, \ldots$

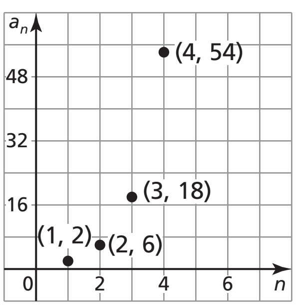
16.

n	1	2	3	4
a_n	2916	972	324	108

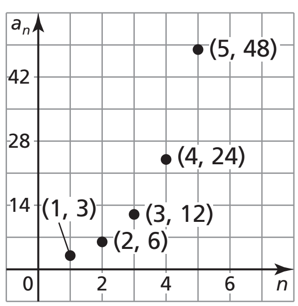
17.

n	1	2	3	4
a_n	11	44	176	704

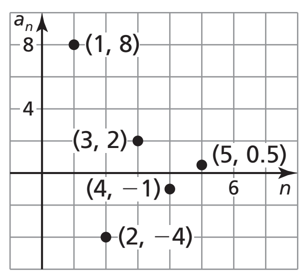
18.



19.



20.



Name____ Date__________

6.6 Recursively Defined Sequences

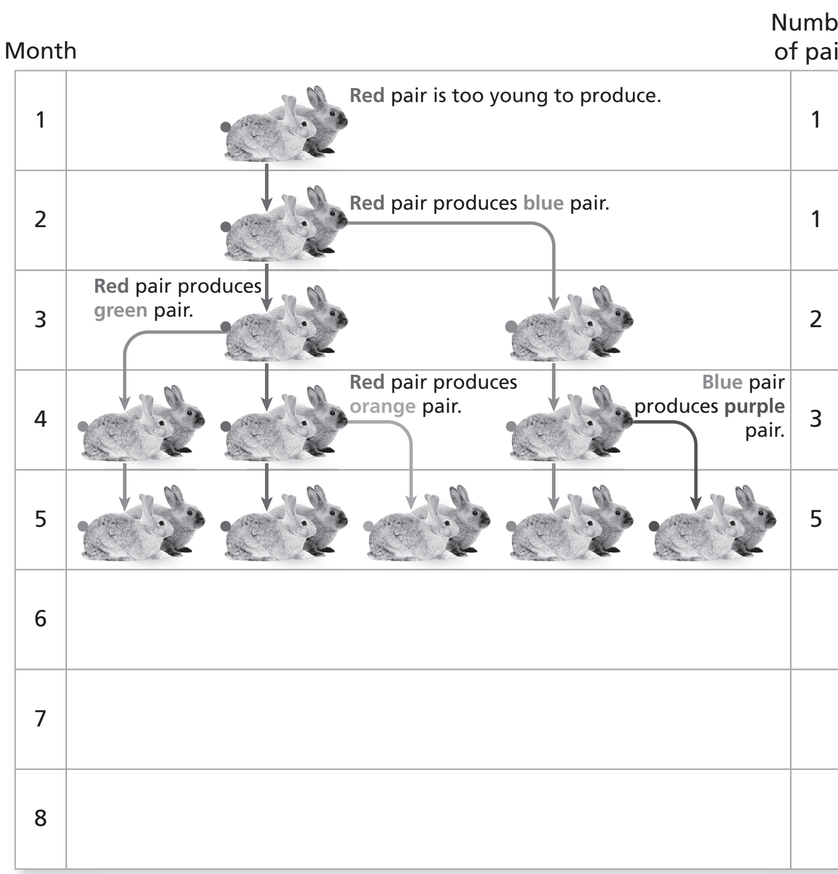
For use with Exploration 6.6

Essential Question How can you define a sequence recursively?

A **recursive rule** gives the beginning term(s) of a sequence and a *recursive equation* that tells how a_n is related to one or more preceding terms

1 EXPLORATION: Describing a Pattern

Work with a partner. Consider a hypothetical population of rabbits. Start with one breeding pair. After each month, each breeding pair produces another breeding pair. The total number of rabbits each month follows the exponential pattern 2, 4, 8, 16, 32, …. Now suppose that in the first month after each pair is born, the pair is too young to reproduce. Each pair produces another pair after it is 2 months old. Find the total number of pairs in months 6, 7, and 8.



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6.6 Recursively Defined Sequences (continued)

2 EXPLORATION: Using a Recursive Equation

Work with a partner. Consider the following recursive equation.

$$a_n = a_{n-1} + a_{n-2}$$

Each term in the sequence is the sum of the two preceding terms.

Complete the table. Compare the results with the sequence of the number of pairs in Exploration 1.

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1						

Communicate Your Answer

3. How can you define a sequence recursively?

4. Use the Internet or some other reference to determine the mathematician who first described the sequences in Explorations 1 and 2.

Name____ Date__________

6.6 Notetaking with Vocabulary
For use after Lesson 6.6

In your own words, write the meaning of each vocabulary term.

explicit rule

recursive rule

Core Concepts

Recursive Equation for an Arithmetic Sequence

$a_n = a_{n-1} + d$, where d is the common difference

Recursive Equation for a Geometric Sequence

$a_n = r \bullet a_{n-1}$, where r is the common ratio

Notes:

Name ____ Date __________

6.6 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–6, write the first six terms of the sequence. Then graph the sequence.

1. $a_1 = -2; a_n = -2a_{n-1}$

2. $a_1 = -4; a_n = a_{n-1} + 3$

3. $a_1 = 4; a_n = 1.5a_{n-1}$

4. $a_1 = 14; a_n = a_{n-1} - 4$

5. $a_1 = -\frac{1}{2}; a_n = -2a_{n-1}$

6. $a_1 = -3; a_n = a_{n-1} + 2$

In Exercises 7 and 8, write a recursive rule for the sequence.

7.

n	1	2	3	4
a_n	324	108	36	12

8.

n	1	2	3	4
a_n	9	14	19	24

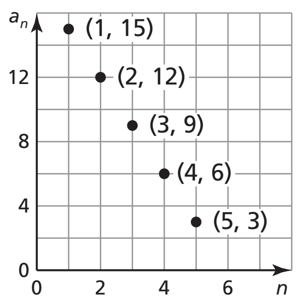
Name____________________ Date__________

6.6 Notetaking with Vocabulary (continued)

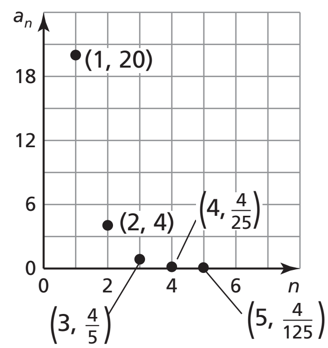
In Exercises 9–13, write a recursive rule for the sequence.

9. $3125, 625, 125, 25, \ldots$ **10.** $8, -24, 72, -216, \ldots$ **11.** $7, 13, 19, 25, \ldots$

12.



13.



In Exercises 14–16, write an explicit rule for the recursive rule.

14. $a_1 = 4; a_n = 3a_{n-1}$ **15.** $a_1 = 6; a_n = a_{n-1} + 11$ **16.** $a_1 = -1; a_n = 5a_{n-1}$

In Exercises 17–19, write a recursive rule for the explicit rule.

17. $a_n = 6n + 2$ **18.** $a_n = (-3)^{n-1}$ **19.** $a_n = -2n + 1$

In Exercises 20–22, write a recursive rule for the sequence. Then write the next two terms of the sequence.

20. $2, 4, 6, 10, 16, 26, \ldots$ **21.** $1, 3, -2, 5, -7, 12, \ldots$ **22.** $1, 2, 2, 4, 8, 32, \ldots$